Supplemental Appendix for "Covariate Balancing and the Equivalence of Weighting and Doubly Robust Estimators of Average Treatment Effects"

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This appendix explains how to obtain IPT propensity score estimates in Stata and R, as well as how to install and use our companion Stata package, `teffects2`. This package implements IPW, AIPW, and IPWRA estimators of the ATE and ATT under unconfoundedness, with several approaches to estimate the weights, including IPT. The package can also be used to estimate the ATT in difference-in-differences settings after a suitable transformation of the outcome variable. Throughout this appendix, as well as in `teffects2`, we restrict our attention to the logit model. In what follows, among other things, we will show how to estimate this model using the method of moments approach of Egel, Graham, and Pinto (2008) and Graham, Pinto, and Egel (2012, 2016) instead of maximum likelihood.

Implementation in Stata

This code illustrates IPT estimation by reproducing the estimate in column 1 of Table 2, which corresponds to the first entry in column 2 of Table 3 in Sant'Anna and Zhao (2020). The parameter of interest is the ATT, and the estimation procedure is based on the sample moment conditions in (2.5). The code can easily be modified for use in other applications.

First, we show how to reproduce this estimate using `teffects2`. To download this package, type

```
ssc install teffects2, all
```

in the Command window. Then, run the following code:

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```
* Load the data
use lalonde, clear

* Restrict attention to the NSW control and CPS comparison units
keep if (dataset == 0 | dataset == 4) & treated == 0

* Recode the NSW control units as "treated" (cf. Smith and Todd, 2005)
replace treated = 1 if dataset == 0

* Specify outcome, treatment, and control variables
local Y diff
local W treated
local X age educ re74 nodegree married black hispanic

* Estimate the ATT using teffects2
teffects2 ipw (‘Y’) (‘W’ ‘X’, ipt), atet
teffects2 aipw (‘Y’ ‘X’) (‘W’ ‘X’, ipt), atet
teffects2 ipwra (‘Y’ ‘X’) (‘W’ ‘X’, ipt), atet
```

As implied by Proposition 3.2, the estimates (and standard errors) obtained with `teffects2 ipw`, `teffects2 aipw`, and `teffects2 ipwra` are identical, except for negligible differences due to floating-point precision. The output also matches the IPT estimate in column 1 of Table 2 as well as the first entry in column 2 of Table 3 in Sant'Anna and Zhao (2020). For example, with `teffects2 ipwra`, we obtain:

```
. teffects2 ipwra (‘Y’ ‘X’) (‘W’ ‘X’, ipt), atet

Treatment effect estimation                     Number of obs     =     16,417
Estimator      : IPW regression adjustment
Outcome model  : linear
Treatment model: logit IPT
------------------------------------------------------------------------------
             |               Robust
        diff | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         ATT |  -901.2702   393.6127    -2.29   0.022    -1672.737   -129.8036
      POmean |   2964.636   254.5088    11.65   0.000     2465.808    3463.464
------------------------------------------------------------------------------
```

In line with Stata's official `teffects` command, which only allows maximum likelihood estimation of the propensity score, `POmean` corresponds to an estimate of the mean untreated outcome (when estimating the ATE) or the mean untreated outcome among the treated (when estimating the ATT, as in the example above).

Second, we show how to obtain the underlying propensity score estimates, $p(\mathbf{X}_i \hat{\gamma}_{0,ipt})$, and how to reproduce the estimate of the ATT from scratch, i.e., without using `teffects2`.

```
* Download the data
use https://tslocz.github.io/lalonde.dta, clear

* Restrict attention to the NSW control and CPS comparison units
keep if (dataset == 0 | dataset == 4) & treated == 0

* Recode the NSW control units as "treated" (cf. Smith and Todd, 2005)
replace treated = 1 if dataset == 0

* Standardize nonbinary covariates
egen age_std = std(age)
egen educ_std = std(educ)
egen re74_std = std(re74)

* Specify outcome, treatment, and control variables
local Y diff
local W treated
local X age_std educ_std re74_std nodegree married black hispanic

* Set up the method of moments procedure
local eq0 (eq0: ((1 - ‘W’) * (1 + exp({that0: ‘X’ _cons})) - 1))
local inst0 instruments(eq0: ‘X’)

* Obtain the IPT propensity score estimates
gmm ‘eq0’, ‘inst0’
predict double xb0, xb equation(that0)
generate double p_hat0 = logistic(xb0)

* Estimate the ATT
generate double term = (p_hat0 * (1 - ‘W’) * ‘Y’) / (1 - p_hat0)
```

```
summarize term
scalar m1_hat = r(mean)
summarize 'Y' if 'W' == 1
scalar m2_hat = r(mean)
summarize 'W'
scalar m3_hat = r(mean)
scalar att = m2_hat - m1_hat / m3_hat
```

The final estimate, implementing the IPW estimator of the ATT with the IPT weights, matches the `teffects2` estimate above, as well as the appropriate estimates in Table 2 and Sant'Anna and Zhao (2020):

```
. display att
-901.27028
```

Although this is not necessary to estimate the ATT, a researcher interested in the ATE also needs to obtain $p(\mathbf{X}_i \hat{\gamma}_{1,ipt})$ using the sample moment conditions in (2.3). To compute these estimates, the code above should be modified as follows:

```
* Set up the method of moments procedure
local eq1 (eq1: ('W' * (1 + exp({that1: 'X' _cons})) / exp({that1:}) - 1))
local inst1 instruments(eq1: 'X')

* Obtain the IPT propensity score estimates
gmm 'eq1', 'inst1'
predict double xb1, xb equation(that1)
generate double p_hat1 = logistic(xb1)
```

In this example, `gmm` fails to converge with default settings, but does converge under some alternative optimization routines. The fact that obtaining $p(\mathbf{X}_i \hat{\gamma}_{1,ipt})$ is more difficult than obtaining $p(\mathbf{X}_i \hat{\gamma}_{0,ipt})$ should be treated as informative rather than problematic, as it reflects the underlying identification challenge—in the LaLonde (1986) data, it is simply very difficult to reweight the experimental subjects to resemble the CPS participants on average.

Implementation in R

As above, we show how to use IPT to reproduce the ATT estimate in column 1 of Table 2, which corresponds to the first entry in column 2 of Table 3 in Sant'Anna and Zhao (2020).

```r
# Install and load the add-on package geex
install.packages("geex")
library(geex)

# Download the data
lalonde <- read.csv("https://tslocz.github.io/lalonde.csv")

# Restrict attention to the NSW control and CPS comparison units
nswcps <- subset(lalonde, (dataset %in% c(0, 4)) & treated == 0)

# Recode the NSW control units as "treated" (cf. Smith and Todd, 2005)
nswcps$W <- as.integer(nswcps$dataset == 0)

# Standardize nonbinary covariates
nswcps$age_std  <- as.numeric(scale(nswcps$age))
nswcps$educ_std <- as.numeric(scale(nswcps$educ))
nswcps$re74_std <- as.numeric(scale(nswcps$re74))

# Specify outcome, treatment, and control variables
Y <- nswcps$diff
W <- nswcps$W
X <- model.matrix(
  ~ age_std + educ_std + re74_std + nodegree + married + black + hispanic,
  data = nswcps
)

# Set up supporting objects
df <- data.frame(W = W, X, check.names = FALSE)
X_cols <- colnames(X)

# Specify starting values
p <- length(X_cols)
gamma_start <- numeric(p)

# Set up the method of moments procedure
score_eq0 <- function(data) {
```

```
  W_i <- data$W
  X_i <- as.vector(as.matrix(data[X_cols]))
  function(theta) {
    eta_i <- sum(X_i * theta)
    p_i   <- plogis(eta_i)
    ((1 - W_i) / (1 - p_i) - 1) * X_i
  }
}

# Obtain the IPT propensity score estimates
mest_eq0 <- m_estimate(
  estFUN = score_eq0,
  data   = df,
  root_control = setup_root_control(start = gamma_start)
)
gamma0  <- as.numeric(coef(mest_eq0))
p_hat0  <- as.vector(plogis(X %*% gamma0))

# Estimate the ATT
term    <- (p_hat0 * (1 - W) * Y) / (1 - p_hat0)
m1_hat  <- mean(term)
m2_hat  <- mean(Y[W == 1])
m3_hat  <- mean(W)
att     <- m2_hat - m1_hat / m3_hat
```

The resulting estimate, implementing the IPW estimator of the ATT with the IPT weights, matches both Stata estimates above, as well as the appropriate estimates in Table 2 and Sant'Anna and Zhao (2020):

```
> att
[1] -901.2702
```

To obtain $p(\mathbf{X}_i\hat{\gamma}_{1,ipt})$, the code above should be modified as follows:

```
# Set up the method of moments procedure
score_eq1 <- function(data) {
  W_i <- data$W
  X_i <- as.vector(as.matrix(data[X_cols]))
```

```
  function(theta) {
    eta_i <- sum(X_i * theta)
    p_i   <- plogis(eta_i)
    (W_i / p_i - 1) * X_i
  }
}

# Obtain the IPT propensity score estimates
mest_eq1 <- m_estimate(
  estFUN = score_eq1,
  data   = df,
  root_control = setup_root_control(start = gamma_start)
)
gamma1  <- as.numeric(coef(mest_eq1))
p_hat1  <- as.vector(plogis(X %*% gamma1))
```

As in Stata, obtaining $p(\mathbf{X}_i\hat{\gamma}_{1,ipt})$ is challenging; however, convergence is achievable with alternative solver choices and better starting values, such as the MLE coefficients.

Appendix References

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LALONDE, R. J. (1986): "Evaluating the Econometric Evaluations of Training Programs with Experimental Data," *American Economic Review*, 76(4), 604–620.

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